

# NAG Toolbox for MATLAB

## f07au

### 1 Purpose

f07au estimates the condition number of a complex matrix  $A$ , where  $A$  has been factorized by f07ar.

### 2 Syntax

```
[rcond, info] = f07au(norm_p, a, anorm, 'n', n)
```

### 3 Description

f07au estimates the condition number of a complex matrix  $A$ , in either the 1-norm or the  $\infty$ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that  $\kappa_\infty(A) = \kappa_1(A^H)$ .

Because the condition number is infinite if  $A$  is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

### 4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

### 5 Parameters

#### 5.1 Compulsory Input Parameters

1: **norm\_p** – string

Indicates whether  $\kappa_1(A)$  or  $\kappa_\infty(A)$  is estimated.

**norm\_p** = '1' or 'O'

$\kappa_1(A)$  is estimated.

**norm\_p** = 'I'

$\kappa_\infty(A)$  is estimated.

*Constraint:* **norm\_p** = '1', 'O' or 'I'.

2: **a(lda,\*)** – complex array

The first dimension of the array **a** must be at least  $\max(1, \mathbf{n})$

The second dimension of the array must be at least  $\max(1, \mathbf{n})$

The  $LU$  factorization of  $A$ , as returned by f07ar.

3: **anorm** – double scalar

If **norm\_p** = '1' or 'O', the 1-norm of the **original** matrix  $A$ .

If **norm\_p** = 'I', the  $\infty$ -norm of the **original** matrix  $A$ .

**anorm** must be computed either **before** calling f07ar or else from a **copy** of the original matrix  $A$ .

*Constraint:* **anorm**  $\geq 0.0$ .

## 5.2 Optional Input Parameters

1: **n** – int32 scalar

*Default:* The second dimension of the array **a**.

*n*, the order of the matrix *A*.

*Constraint:*  $n \geq 0$ .

## 5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, rwork

## 5.4 Output Parameters

1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of *A*. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, *A* is singular to working precision.

2: **info** – int32 scalar

**info** = 0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** =  $-i$

If **info** =  $-i$ , parameter *i* had an illegal value on entry. The parameters are numbered as follows:

1: **norm\_p**, 2: **n**, 3: **a**, 4: **lda**, 5: **anorm**, 6: **rcond**, 7: **work**, 8: **rwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

## 7 Accuracy

The computed estimate **rcond** is never less than the true value  $\rho$ , and in practice is nearly always less than  $10\rho$ , although examples can be constructed where **rcond** is much larger.

## 8 Further Comments

A call to f07au involves solving a number of systems of linear equations of the form  $Ax = b$  or  $A^H x = b$ ; the number is usually 5 and never more than 11. Each solution involves approximately  $8n^2$  real floating-point operations but takes considerably longer than a call to f07as with one right-hand side, because extra care is taken to avoid overflow when *A* is approximately singular.

The real analogue of this function is f07ag.

## 9 Example

```
norm_p = '1';
a = [complex(-3.29, -2.39), complex(-1.91, +4.42), complex(-0.14, -1.35),
     complex(1.72, +1.35);
      complex(0.2376120269469406,      +0.2559596521570857),
     complex(4.895180634002975, ...
```

```
-0.711362223485444), complex(-0.462279846639494, +1.696610587680362),  
...  
    complex(1.226852844063328, +0.6189731619114427);  
    complex(-0.1019520808892006, -0.7010135339437114), complex(-  
0.6691496431943212, ...  
        +0.3688698547971655),    complex(-5.141410913810232, -  
1.129969734660953), ...  
    complex(0.9982579915199449, +0.3850152275763778);  
    complex(-0.5358546703595748, +0.2707272529359829), complex(-  
0.2040217794587079, ...  
        +0.8601180679984047),    complex(0.008233047063940025,  
+0.1210636819910635), ...  
    complex(0.1482391810748419, -0.1252239986075848)];  
anorm = 12.88382186360408;  
[rcond, info] = f07au(norm_p, a, anorm)
```

```
rcond =  
    0.0066  
info =  
    0
```